1 Laboratory: Diffraction and Imaging

References: Optics, E. Hecht, Chapter 10; Introduction to Optics, Pedrotti & Pedrotti, Chapters 16, 18

We are able to determine locations of images and their magnifications by using the concept of light as a “ray” (geometrical optics). However, the concept of light as a “wave” also is fundamental to imaging, particularly in its manifestation in “diffraction”, which is the fundamental limitation on the action of an optical imaging system. “Interference” and “diffraction” may be interpreted as the same phenomenon, differing only in the number of sources involved (interference \(\Rightarrow\) few sources, say 2 - 10; diffraction \(\Rightarrow\) many sources, up to an infinite number). The theory of diffraction leads to the theoretical limit to the performance of an imaging system.

1.1 Theory

This lab will investigate the diffraction patterns generated from apertures of different shapes and observed at different distances. As we have mentioned, the physical process that results in observable diffraction patterns is identical to that responsible for interference. In the latter case, we generally speak of the intensity patterns generated by light after passing through a few apertures whose size(s) generally are smaller than the distance between them. The term *diffraction* usually is applied to the process either for a single large aperture, or (equivalently) a large number of small (usually infinitesimal) contiguous apertures.

In studies of both interference and diffraction, the patterns are most obvious if the illumination is *coherent*, which means that the phase of the sinusoidal electric fields is rigidly deterministic. In other words, knowledge of the phase of the field at some point in space and/or time determines the phase at other points in space and/or time. Coherence has two flavors: spatial and temporal. For *spatially coherent* light, the phase difference \(\Delta \phi \equiv \phi_1 - \phi_2\) of the electric field measured at two different points at the same time at two points in space separated by a vector distance \(\Delta r\) remains constant for all times and for all such points in space. If the phase difference measured at the SAME location at two different times separated by \(\Delta t \equiv t_1 - t_2\) is the same for all points in space, the light is *temporally coherent*. Light from a laser may be considered to be BOTH spatially and temporally coherent. The properties of coherent light allow phase differences of light that has traveled different paths to be made visible, since the phase difference is constant with time. In interference, the effect often results in a sinusoidal fringe pattern in space. In diffraction, the phase difference of light from different points in the same large source can be seen as a similar pattern of dark and bright fringes, though not (usually) with sinusoidal spacing.

Observed diffraction patterns from the same object usually look very different at different distances to the observation plane. If viewed very close to the aperture (in the *Rayleigh-Sommerfeld* diffraction region), then Huygens’ principle says that the amplitude of the electric field is the summation (integral) of the spherical wavefronts generated by each point in the aperture. The resulting amplitude pattern may be quite complicated to evaluate. If observed somewhat farther from the aperture, the spherical wavefronts may be accurately approximated by paraboloidal wavefronts. The approximation applies in the *near field*, or the *Fresnel diffraction region*. If viewed at a large distance compared to the extent of the object, the light from different locations in the aperture may be accurately modeled as *plane waves* with different wavefront tilts. This occurs in the *Fraunhofer diffraction region*.

1.1.1 Fresnel Diffraction

In the Fresnel diffraction region (where the distance between the object and the observation plane is small compared to the size of the object). The diffraction pattern resembles the original object with “fuzzy” or “ringing” edges. If the size of the object is increased, so will be the Fresnel diffraction pattern. If the distance between the object and the observation is increased, though still within
the Fresnel diffraction region, the size of the “ringing” artifacts increases. In general, the Fresnel diffraction pattern of an object \( f[x, y] \) observed at a distance \( z_1 \) “downstream” is proportional to the expression:

\[
g[x, y] = \frac{1}{i\lambda_0 z_1} \exp \left[ -\frac{2\pi i z_1}{\lambda_0} \right] \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f[x - \alpha, y - \beta] \exp \left[ -\frac{i\pi (\alpha^2 + \beta^2)}{\lambda_0 z_1} \right] d\alpha d\beta
\]

\[
= \frac{1}{i\lambda_0 z_1} \exp \left[ -\frac{2\pi i z_1}{\lambda_0} \right] \left( f[x, y] * \exp \left[ -\frac{i\pi (x^2 + y^2)}{\lambda_0 z_1} \right] \right)
\]

\[
= f[x, y] * \left( \frac{1}{i\lambda_0 z_1} \exp \left[ +\frac{2\pi i z_1}{\lambda_0} \right] \exp \left[ -\frac{i\pi (x^2 + y^2)}{\lambda_0 z_1} \right] \right)
\]

where \( h[x, y; z_1] \) is the “impulse response of light propagation” for wavelength \( \lambda_0 \) and axial distance \( z_1 \). The calculation has the form of the mathematical operation of convolution of the object pattern and a quadratic-phase pattern (which represents the paraboloidal shape of the individual waves). The convolution operation involves a translation of the reversed input function in the integration coordinates \( [\alpha, \beta] \), followed by multiplication by the quadratic-phase factor \( \exp \left[ -\frac{i\pi (\alpha^2 + \beta^2)}{\lambda_0 z_1} \right] \) and then evaluation of the area for each value of the output coordinates \( [x, y] \). Thus it is “complicated” and computationally intensive.

A 1-D model of Fresnel diffraction for square input apertures with different widths is shown in the figure:

Profiles of Fresnel diffraction patterns of the “slit” functions \( RECT \left[ \frac{x}{b} \right] \cdot 1 \left[ y \right] \), for two values of \( b \), showing that the “width” of the output pattern increases in proportion to the “width” of the input pattern.

1.1.2 Fraunhofer Diffraction

At large distances from the object plane, the diffraction is in the far field or Fraunhofer diffraction region. Here, the pattern of diffracted light usually does not resemble the object at all. The size of the observed pattern varies in proportion to the reciprocal of the object dimension, i.e., the larger the object, the smaller the diffraction pattern. Note that increasing the size of the object also produces a brighter diffraction pattern, because more light reaches the observation plane. The mathematical relation between the shape and size of the output relative to that of the input is a Fourier transform, which is a mathematical coordinate transformation that was “discovered” by Baron Jean-Baptiste
Joseph de Fourier in the early 1800s. For the same input pattern $f[x,y]$, the diffraction pattern in the Fraunhofer region has the form:

\[
\mathcal{F}_2 \{f[x,y]\} \equiv F[\xi,\eta] = \int_{-\infty}^{+\infty} f[x,y] (\exp[+2\pi i (\xi x + \eta y)])^* \, dx \, dy
\]

\[
= \int_{-\infty}^{+\infty} f[x,y] \exp[-2\pi i (\xi x + \eta y)] \, dx \, dy
\]

In words, the input function $f[x,y]$ is transformed into the equivalent function $F[\xi,\eta]$, where the coordinates $\xi,\eta$ are spatial frequencies measured in cycles per unit length, e.g., cycles per mm. In optical propagation, the end result is a function of the original 2-D coordinates $[x,y]$, which means that the coordinates $[\xi,\eta]$ are “mapped” back to the space domain via a scaling factor. Since the coordinates of the transform have dimensions of (length)$^{-1}$ and the coordinates of the diffraction light have dimensions of length, the scale factor applied to $\xi$ and $\eta$ must have dimensions of (length)$^2$. It is easy to show that the scaling factor is the product of the two length parameters available in the problem: the wavelength $\lambda_0$ and the propagation distance $z_1$. The pattern of diffracted light in the Fraunhofer diffraction region is:

\[
g[x,y] \propto \mathcal{F}_2 \{f[x,y]\}|_{\lambda_0 z_1} \equiv \int_{-\infty}^{+\infty} f[\alpha,\beta] \exp \left[ -2\pi i \left( \frac{x}{\lambda_0 z_1} + \frac{y}{\lambda_0 z_1} \right) \right] \, d\alpha \, d\beta
\]

In mathematical terms, this is a “linear, shift-variant” operation; it is linear because if the input amplitude is scaled by a constant factor, the output amplitude is scaled by the same factor. It is shift variant because a translation of the input does not produce a corresponding transformation of the output. Because Fraunhofer diffraction is shift variant, it may NOT be represented as a single convolution. However, once the Fourier transform is understood, it is very easy to visualize Fraunhofer diffraction patterns of many kinds of objects.

The study of Fourier transforms allows us to infer some important (and possibly counterintuitive) properties of Fraunhofer diffraction:

1. Scaling Theorem: If the scale factor of the aperture function $f[x,y]$ increases, then the resulting diffraction pattern becomes brighter and “smaller,” i.e., the scale factor is proportional to the reciprocal of the scale factor of the input function.

2. Shift Theorem: Translation of $f[x,y]$ adds a linear term to the phase of the diffraction pattern, which is not visible in the irradiance. Thus translation of the input has no visible effect on the diffraction pattern.

3. Modulation Theorem: If an aperture can be expressed as the product of two functions, the amplitude of the diffraction pattern is their convolution.

4. Filter Theorem: if an aperture pattern is the convolution of two patterns, the amplitude of the resulting diffraction pattern is the product of the amplitudes of the individual patterns.

**Example:** Consider Fraunhofer diffraction of a simple 2-D rectangular object:

\[
f[x,y] = RECT \left( \frac{x}{a}, \frac{y}{b} \right) = \begin{cases} 1 & \text{if } x = \frac{a}{2} \text{ and } |y| < \frac{b}{2} \\ \frac{1}{4} & \text{if } |x| < \frac{a}{2} \text{ or } |y| < \frac{b}{2} \\ \frac{1}{4} & \text{if } |x| = \frac{a}{2} \text{ and } |y| = \frac{b}{2} \\ 0 & \text{if } |x| > \frac{a}{2} \text{ and } |y| > \frac{b}{2} \end{cases}
\]

The integral evaluates rather easily:

\[
g[x,y] \propto \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} RECT \left( \frac{\alpha}{a}, \frac{\beta}{b} \right) \exp \left[ -\frac{2\pi i (x\alpha + y\beta)}{\lambda_0 z} \right] \, d\alpha \, d\beta
\]
\[
\int_{y=+\frac{b}{2}}^{y=-\frac{b}{2}} \int_{x=-\frac{a}{2}}^{x=+\frac{a}{2}} \exp\left[-\frac{2\pi ix\alpha}{\lambda_0 z}\right] \exp\left[-\frac{2\pi iy\beta}{\lambda_0 z}\right] \, d\alpha \, d\beta
\]

\[
= \int_{x=-\frac{a}{2}}^{x=+\frac{a}{2}} \exp\left[-\frac{2\pi ix}{\lambda_0 z}\right] \alpha \cdot \int_{y=-\frac{b}{2}}^{y=+\frac{b}{2}} \exp\left[-\frac{2\pi iy}{\lambda_0 z}\right] \beta \, d\alpha \, d\beta
\]

\[
= \exp\left[-\frac{2\pi ia}{\lambda_0 z}\right] \alpha \cdot \exp\left[-\frac{2\pi ib}{\lambda_0 z}\right] \beta \cdot \left[\frac{\sin \left(\frac{\pi ax}{\lambda_0 z}\right)}{\frac{\pi ax}{\lambda_0 z}} \cdot \frac{\sin \left(\frac{\pi by}{\lambda_0 z}\right)}{\frac{\pi by}{\lambda_0 z}}\right]
\]

\[
= |a| \left(\frac{\sin \left(\frac{\pi ax}{\lambda_0 z}\right)}{\frac{\pi ax}{\lambda_0 z}}\right) \cdot |b| \left(\frac{\sin \left(\frac{\pi by}{\lambda_0 z}\right)}{\frac{\pi by}{\lambda_0 z}}\right) \equiv |ab| SINC \left[\frac{x}{\lambda_0 z a}, \frac{y}{\lambda_0 z b}\right]
\]

\[
g[x, y] \propto (ab)^2 \left( SINC \left[\frac{x}{\lambda_0 z a}, \frac{y}{\lambda_0 z b}\right]\right)^2
\]

Where the 2-D “SINC” function is defined as the orthogonal product of two 1-D SINC functions:

\[
SINC [x, y] \equiv SINC [x] \cdot SINC [y] \equiv \frac{\sin [\pi x]}{\pi x} \cdot \frac{\sin [\pi y]}{\pi y}
\]

which has the pattern shown in the figure.

### 1.1.3 Fraunhofer Diffraction in Optical Imaging Systems

A monochromatic point object located a long distance away from an imaging system produces a set of wavefronts that are “regularly” spaced (separated by the wavelength) and are approximately planar. The entrance pupil of the optical system (the image of the aperture stop in object space) collects a section of the plane wavefront and the optical elements convert it to a spherical wave that converges to an image “point.” We can use the concept of Fraunhofer diffraction to define the “angular resolution” of the imaging system.

As an introduction, consider an optical system that consists of only the entrance pupil (which coincides with the aperture stop because no other optics are involved), as shown in the figure. The “pieces” of the object wavefronts that are collected by the stop will continue to propagate “downstream.” If observed a long distance from the stop, the irradiance would be the Fraunhofer diffraction pattern of the stop; the smaller the stop, the larger the diffraction pattern and vice versa.

Of course, the observed irradiance is the time average of the squared magnitude of the amplitude, and thus is nonnegative.
The system may be modeled in three stages:

1. Propagation from the input object $f(x,y)$ to the Fraunhofer diffraction region over the distance $z_1$.
2. Multiplication by the (possibly complex-valued) transmittance function $t(x,y) = |t(x,y)| \exp [i \Phi_t(x,y)]$ that specifies the aperture (or pupil), and
3. A second propagation over the distance $z_2$ into the Fraunhofer diffraction region (determined from the aperture).

To eliminate an awkward notation, we will substitute the notation $p(x,y)$ for the magnitude of the pupil function $|t(x,y)|$. In this example, we assume that the pupil has no phase component, so that $\Phi_t(x,y) = 0$, though solution of the more general case is straightforward. The 2-D input function $f(x,y; z = 0)$ is illuminated by a unit amplitude monochromatic plane wave with wavelength $\lambda_0$. The light propagates into the Fraunhofer diffraction region at a distance $z_1$, where the resulting amplitude pattern is:

$$E(x,y; z_1) = \frac{\mathcal{E}_0}{i\lambda_0 z_1} \exp \left[ +2\pi i \frac{z_1}{\lambda_0} \right] \exp \left[ +i\pi \frac{(x^2 + y^2)}{\lambda_0 z_1} \right] F \left[ \frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1} \right]$$

This pattern illuminates the 2-D aperture function $p(x,y)$ and then propagates the distance $z_2$ into the Fraunhofer diffraction region (determined by the support of $p$). A second application produces the amplitude at the observation plane:

$$E(x,y; z_1 + z_2) = \mathcal{E}_0 \left( \frac{1}{i\lambda_0 z_1} e^{+2\pi i \frac{z_1}{\lambda_0}} e^{+i\pi \frac{(x^2 + y^2)}{\lambda_0 z_1}} \right) \left( \frac{1}{i\lambda_0 z_2} e^{+2\pi i \frac{z_2}{\lambda_0}} e^{+i\pi \frac{(x^2 + y^2)}{\lambda_0 z_2}} \right)$$

$$\cdot \mathcal{F}_2 \left( F \left[ \frac{x}{\lambda_0 z_1}, \frac{y}{\lambda_0 z_1} \right], p(x,y) \right) \bigg|_{\xi = \frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}}$$
\[ E_0 \left( -\frac{1}{\lambda_0^2 z_1 z_2} \right) e^{\frac{2\pi i z_1 z_2}{\lambda_0}} e^{i\pi \left( \frac{x^2 + y^2}{\lambda_0} \right) \left( \frac{1}{z_1} + \frac{1}{z_2} \right)} \cdot (\lambda_0 z)^2 (f - \lambda_0 z_1 \xi, -\lambda_0 z_2 \eta) * P[\xi, \eta] |_{\xi = \frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2}} \]

\[ = E_0 \left( \frac{z_1}{z_2} \right) e^{\frac{2\pi i z_1 z_2}{\lambda_0}} e^{i\pi \left( \frac{x^2 + y^2}{\lambda_0} \right) \left( \frac{1}{z_1} + \frac{1}{z_2} \right)} \left( f \left[ -\frac{z_1}{z_2} x, -\frac{z_1}{z_2} y \right] \right) * P \left[ \frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2} \right] \]

\[ = \frac{E_0}{M_T} e^{\frac{2\pi i z_1 z_2}{\lambda_0}} e^{i\pi \left( \frac{x^2 + y^2}{\lambda_0} \right) \left( \frac{1}{z_1} + \frac{1}{z_2} \right)} \left( f \left[ \frac{x}{M_T}, \frac{y}{M_T} \right] \right) * P \left[ \frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2} \right] \]

where the theorems of the Fourier transform and the definition of the transverse magnification from geometrical optics, \( M_T = -\frac{z_2}{z_1} \), have been used. Note that if the propagation distances \( z_1 \) and \( z_2 \) must both be positive in Fraunhofer diffraction, which requires that \( M_T < 0 \) and the image is “reversed.”

The irradiance of the image is proportional to the squared magnitude of the amplitude:

\[ |E[x, y; z_1 + z_2]|^2 = \left| \frac{E_0}{M_T} \right|^2 \left| f \left[ \frac{x}{M_T}, \frac{y}{M_T} \right] \right| * P \left[ \frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2} \right] \]

In words, the output amplitude created by this imaging “system” is the product of some constants, a quadratic-phase function of \([x, y]\), and the convolution of the input amplitude scaled by the transverse magnification and the scaled replica of the spectrum of the aperture function, \( P \left[ \frac{x}{\lambda_0 z_2}, \frac{y}{\lambda_0 z_2} \right] \). Since the output is the result of a convolution, we identify the spectrum as the impulse response of a shift-invariant convolution that is composed of two shift-variant Fourier transforms and multiplication by a quadratic-phase factor of \([x, y]\). This system does not satisfy the strict conditions for shift invariance because of the leading quadratic-phase factor and the fact that the input to the convolution is a scaled and reversed replica of the input to the system. That said, these details are often ignored to allow the process to considered to be shift invariant. We will revisit this conceptual imaging system after considering the mathematical models for optical elements.

We can apply the observation that the Fraunhofer diffraction pattern is proportional to the Fourier transform of the 2-D input distribution \( f[x, y] \) to perform useful filtering on an input image by adding a second lens to compute the “Fourier transform of the Fourier transform.” The most obvious way to do this is the “4f correlator:”

\[ \text{Apparatus for viewing Fraunhofer diffraction patterns} \]

The second lens is located one focal length away from the Fourier transform plane and the output is observed one focal length away from the lens. It is easy to trace a ray from an “arrow” located at the object plane parallel to the axis. The “image” of this ray will be inverted (“upside down”),
which indicates that the “Fourier transform of the Fourier transform” is a reversed replica of the function:

\[ \mathcal{F}_2 \{ \mathcal{F}_2 \{ f(x, y) \} \} = \mathcal{F}_2 \left\{ F \left[ \frac{x}{\lambda_0 f}, \frac{y}{\lambda_0 f} \right] \right\} \propto f(-x, -y) \]

Demonstration that the output of the 4f-system is a reversed replica \( f(-x, -y) \) of the input function \( f(x, y) \).

This imaging system makes the Fourier transform \( F[\xi, \eta] \) of the input function \( f[x, y] \) “accessible” where it can be modified.
1.2 Equipment:
1. He:Ne laser
2. microscope objective to expand the beam; larger power gives larger beam in shorter distance;
3. pinhole aperture to “clean up” the beam;
4. positive lens with diameter $d \approx 50 \text{mm}$ and focal length $f \lesssim 600 \text{mm}$, to collimate the beam;
5. positive lens with diameter $d \approx 50 \text{mm}$ and focal length $f > 200 \text{mm}$, to compute the Fourier transform;
6. aluminum foil, needles, and razor blades to make your own objects for diffraction;
7. set of Metrologic transparencies;
8. digital camera to record diffraction patterns.

1.3 Procedures:

1.3.1 Fresnel Diffraction
1. Set up the experimental bench as in the figure with the observing screen close to the aperture (within a foot or so) to examine the results in the Fresnel diffraction region. Measure and record the relevant distances. A number of apertures are available for use, including single and multiple slits of different spacings, single and multiple circular apertures, needles (both tips and eyes), razor blades, etc. In addition, aluminum foil and needles are available to make your own apertures.

![Apparatus for viewing Fresnel diffraction patterns (and Fraunhofer patterns if $z_1$ is sufficiently large).](image)

(a) Begin with a single slit, a square aperture, or a circular aperture. Note the form of the diffraction pattern. For example, sketch how its “brightness” changes with position and note the sizes and locations of any features. For a slit or circular aperture, you should note light and dark regions in the pattern; measure the positions of some maxima and minima (at least 5). Use the data to derive a scale of the pattern. Sketch the pattern noting the scale.

(b) Repeat the previous step with a “wider” slit or aperture. Note the difference in the results.

(c) Vary the distance between the screen and the diffracting object. Repeat measurements. What is the relation between the change in distance and the change in scale of the pattern? Repeat for 5 different distances where the character of the pattern remains the same.
(d) Repeat the procedure with a knife edge ($f [x, y] = STEP [x] \cdot 1 [y]$) as the object. Sketch the pattern observed. You will see that the intensity distribution near the edge of the geometric shadow is not a sharp transition, but rather an undulatory pattern; a magnifying lens, microscope, or digital camera may be helpful to view the pattern, but BE SURE THAT THE LASER LIGHT HAS BEEN ATTENUATED SUFFICIENTLY.

(e) Photograph the Fresnel diffraction pattern from the knife edge at several distances $z_2$. Describe the qualitative differences in the patterns at the different distances.

1.3.2 Fraunhofer Diffraction

1. Now observe the diffraction pattern far from the aperture (several feet away for a small aperture, a proportionally larger distance for a larger aperture) to examine Fraunhofer diffraction. You may “fold” the pattern with one or two mirrors or you may use a lens to “image” the pattern, i.e., to bring the image of the pattern created “a long distance away” much closer to the object. Whichever method you use, be sure to use the same setup for all measurements.

(a) For an aperture of a known fixed (small) size, increase the distance to the observation plane as much as you can. Estimate the location of the transition between the Fresnel and Fraunhofer diffraction regions (this will certainly be ill-defined and “fuzzy”). Record and justify your measurement.

(b) Add another lens to the system as shown below to “bring infinity closer”

i. Set up the “spatial filter” consisting of a microscope objective (or positive lens with a short focal length of about $f \approx 15 \text{ mm}$) and a “pinhole” that is large enough to pass the main beam but small enough to block stray light.

ii. Add a positive lens after the input transparency with a large aperture and focal length of about $f \approx 150 – 200 \text{ mm}$. Ideally, the lens should be located one focal length after the transparency, though this is not critical for the current application. The observation plane should be located one focal length after the lens, which is the location of the smallest image.

(c) Observe Fraunhofer diffraction from apertures of the same shape but different sizes, especially the circular apertures on Slides #17 and #18. Measure the size of observable features and repeat this measurements for the different sizes. Determine the influence of the physical dimension of the apertures on the diffraction pattern. This is the source of the resolution parameters for optical systems with different-size pupils.

(d) Repeat the procedure using a periodic structure (diffraction grid or grating) as the object. Among these, sketch the diffraction patterns of specific transparencies available in the Metrologic set, including:
• #4 (parallel lines with wide spacing)
• #5 (parallel lines with medium spacing)
• #6 (parallel lines with narrow spacing)
• #7 (concentric circles with wide spacing)
• #8 (concentric circles with medium spacing)
• #9 (concentric circles with narrow spacing)
• #10 (crossed" gratings, wide spacing)
• #11 (crossed" gratings, medium spacing)
• #12 (crossed" gratings, narrow spacing).

(e) Now overlay a periodic structure (grid) with a circular aperture and observe the pattern. The overlaying of the two slides produces the product of the two patterns (also called the modulation of one pattern by the other).

(f) Examine the image and diffraction pattern of the transparency Albert (Metrologic slide #24). Note the features of the diffraction pattern and relate them to the features of the transparency.

(g) Examine the pattern generated by a Fresnel Zone Plate (Metrologic slide #13) at different distances. The FZP is a circular grating whose spacing decreases with increasing distance from the center. Sketch a side view of the FZP and indicate the diffraction angle for light incident at different distances from the center of symmetry. You might also overlap another transparency (such as a circular aperture) and the FZP and record the result. I guarantee that this result will not resemble that of part d.

(h) Examine and photograph the Fraunhofer diffraction patterns of other objects, such as the tip and/or the eye of the needle, or you can make your own objects using the aluminum foil; Photograph the objects and the resulting Fraunhofer diffraction patterns. For example, try to make two holes close together of about the same size. Observe the pattern. Repeat after enlarging these same holes and after creating new holes somewhat farther apart. Relate the observations to the laboratory on interference by division of wavefront.

1.3.3 Optical Filtering

1. Set up the experimental bench to see Fraunhofer diffraction with a lens, so that the output is the Fourier transform of the input. Then add a second lens $L_2$ to compute the Fourier transform of the Fourier transform.

2. Make a pinhole aperture with a needle and a white index card. This pinhole will be used to position lens $L_2$. Place the pinhole at the Fourier transform plane made by the first lens.
3. Place lens \( L_2 \) one focal length from the Fourier transform plane and a mirror one focal length behind the lens. Look at the image of the pinhole on the rear side of the index card while moving lens \( L_2 \) along the optical axis. The correct location of lens \( L_2 \) occurs where the image of the pinhole is smallest. Remove the pinhole without moving the holder for the pinhole; this is the location of the Fourier transform plane.

4. Replace the mirror by a viewing screen and insert a white light source as shown:

The image of the transparency should be in focus.

5. Observe the images of the following Metrologic slides: \#10 (medium grid), \#13 (concentric circles with variable widths; this is a Fresnel zone plate); \#19 (fan pattern).

6. Put Metrologic slide \#10 (medium grid) at the input plane. The slides \#3 (circular aperture), \#15 (narrow slit), and a square aperture from \#16 will be used as filters placed at the Fourier transform plane. You also may want to use a small pinhole as a filter; pierce a piece of aluminum foil with a needle and place at the Fourier transform plane.

(a) With no filter, observe and/or photograph the output.

(b) For the medium grid, allow only the central “dot” to pass; observe and/or photograph the output.

(c) Allow the other dots to pass, one at a time; observe and/or photograph the output.

(d) Allow the central \( 3 \times 3 \) set of nine dots to pass; observe and/or photograph the output.
(e) Allow the central vertical row of dots to pass; observe and/or photograph the output.
(f) Allow the central horizontal row of dots to pass; observe and/or photograph the output.
(g) Allow an off-center horizontal row of dots to pass; observe and/or photograph the output.
(h) Allow a diagonal row of dots to pass; observe and/or photograph the output.

7. Use the same setup, but replace the input with Metrologic slide #7 (concentric wide circles)
   (a) With no filter, observe and/or photograph the output.
   (b) Use the horizontal slit (slide #15) to allow part of the diffracted light to pass; observe and/or photograph the output.

8. Use slide #22 (simulation of cloud-chamber photograph) as the object and slide #26 (transparent bar with obstruction at center) as the filter. Position the filter so that the transparent bar is perpendicular to the lines in the input.

9. Use slide #25 as the input and #26 as the filter.
   (a) Orient the filter bar in the vertical direction; observe the output.
   (b) Orient the filter bar in the horizontal direction; observe the output.

10. Use slide #24 (halftone image of Albert Einstein) as the input and a variable-diameter iris or slides #17 and #18 (circular apertures) to make a lowpass filter. Photograph the results.

2 Questions

1. This experiment demonstrates that interaction of light with an obstruction will spread the light. For example, consider Fresnel diffraction of two identical small circular apertures that are separated by a distance d. How will diffraction affect the ability to distinguish the two sources? Comment on the result as lens diameter d is made smaller.

2. The Fresnel Zone Plate (Metrologic slide #13) may be viewed as a circularly symmetric grating with variable period that decreases in proportion to the radial distance from the center. It is possible to use the FZP as an imaging element (i.e., as a lens). Use the model of diffraction from a constant-period grating to describe how the FZP may be used to “focus” light in an optical imaging system. This may be useful for wavelengths (such as x rays) where imaging lenses do not exist.

3. Use the observations of the Fraunhofer diffraction patterns of circular apertures to explain the concept of “resolution” as it applies to imaging systems.